

Calculus Summer Packet

Work the following problems in the space provided. Show all necessary work.

Algebra

Exponents.

1)
$$\frac{(8x^3yz)^{1/3}(2x)^3}{4x^{1/3}(yz^{2/3})^{-1}}$$

Factor Completely.

2) $9x^2 + 3x - 3xy - y$

3) $64x^6 - 1$

4) $42x^4 + 35x^2 - 28$

5) $15x^{5/2} - 2x^{3/2} - 24x^{1/2}$

Rationalize the denominator / numerator.

6)
$$\frac{x}{1 - \sqrt{x-2}}$$

7)
$$\frac{\sqrt{x+1}+1}{x}$$

Simplify the rational expression.

8)
$$\frac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4}$$

Solve algebraic equations and inequalities.

Use synthetic division to help factor the following, state all factors and roots.

9) $p(x) = x^3 + 4x^2 + x - 6$	10) $p(x) = 6x^3 - 17x^2 - 16x + 7$
11) $p(x) = 25x^3 + 50x^2 - 9x - 18$	12) $p(x) = x^4 - x^3 - 2x - 4$
13) <u>Explain</u> why $\frac{3}{2}$ cannot be a root of $4x^5 + cx^3 - dx + 5 = 0$, where c and d are integers.	14)) <u>Explain</u> why $x^4 + 7x^2 + x - 5 = 0$ must have a root in the interval $[0, 1]$, ($0 \leq x \leq 1$)

Solve.

15) $(x + 3)^2 > 4$	16) $\frac{x+5}{x-3} \leq 0$	17) $x^3 + 2x^2 - 3x \leq 0$
18) $\frac{x^2 - 9}{x + 1} \geq 0$	19) $0 < x - 2 < 8$	20) $ 5x - 1 > 9$

Solve the system.

21) $x - y + 1 = 0$ $y - x^2 = -5$	22) $x^2 - 4x + 3 = y$ $-x^2 + 6x - 9 = y$
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Graphing and Functions

Linear Graphs - Write the equation of the line described below.

23) Passes through the point (2, -1) and has slope -1/3.	24) Passes through the point (4, -3) and is perpendicular to $3x + 2y = 4$.	25) Passes through (-1, -2) and is parallel to $y = \frac{3}{5}x - 1$.
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Conic Sections - Write the equation in standard form and identify the conic.

26) $x = 4y^2 + 8y - 3$	27) $x^2 + y^2 - 4x + 2y - 4 = 0$
28) $4x^2 - 16x + 3y^2 + 24y + 52 = 0$	29) $x^2 - 4y^2 + 2x + 24y - 19 = 0$

Functions

Find the domain and range of the following.

domain restrictions - denominator $\neq 0$, argument of a log or $\ln > 0$, radicand of even index must be ≥ 0
range restrictions- reasoning, if all else fails, use graphing calculator

30) $y = \frac{3}{x-2}$	31) $y = \log(x-3)$
32) $y = x^4 + x^2 + 2$	33) $y = \sqrt{2x-3}$
34) $y = x-5 $	35) Given $f(x)$ below, graph over the domain $[-3, 3]$. What is the range? $f(x) = \begin{cases} x & \text{if } x \geq 0 \\ 1 & \text{if } -1 \leq x < 0 \\ x-2 & \text{if } x < -1 \end{cases}$

Compositions and Inverses - Find the compositions and inverses as indicated below.

Let: $f(x) = x^2 + 3x - 2$ $g(x) = 4x - 3$ $h(x) = \ln x$ $w(x) = \sqrt{x-4}$			
36) $g^{-1}(x)$	37) $h^{-1}(x)$	38) $w^{-1}(x)$, for $x \geq 4$	39) $f(g(x))$
40) $h(g(f(1)))$	41) $g(h(x))$	42) $g(w(x))$	43) Does $y = 3x^2 - 9$ have an inverse function? Explain your answer.

Basic Shapes of Curves:

Sketch the graphs on separate paper. You may use your graphing calculator to verify the graph, but you should be able to graph the following by knowledge of the shape of the curve, by plotting a few points, and by your knowledge of transformations.

44) $y = \sqrt{x}$	45) $y = \ln x$	46) $y = x - 2 $
47) $y = \frac{1}{x}$	48) $y = \frac{1}{x-2}$	49) $y = \frac{x}{x^2-4}$
50) $y = e^x$	51) $y = 3^x$	52) $f(x) = \begin{cases} \sqrt{25-x^2} & \text{if } x < 0 \\ \frac{x^2-25}{x-5} & \text{if } x \geq 0, x \neq 5 \\ 0 & \text{if } x = 5 \end{cases}$

Asymptotes – Identify any asymptotes.

53) $y = \frac{1}{x-1}$	54) $x^2 - y^2 = 1$	55) $y = \frac{x^2}{x^2-1}$
56) $y = \frac{3x^2}{2x^2-3x+3}$	57) $y = \frac{5x^2-5x+1}{x-1}$	58) $y = \frac{2x^2}{3x^3-4x+1}$

Even and Odd Functions - Identify as odd, even, or neither. Show substitutions!

Even: $f(x) = f(-x)$ Odd: $f(-x) = -f(x)$

59) $f(x) = x^3 + 3x$	60) $f(x) = x^4 - 6x^2 + 3$	61) $f(x) = \frac{x^3 - x}{x^2}$	62) $f(x) = \sin 2x$
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Test for symmetry. Show substitutions.

Symmetric to y axis: replace x with - x and relation remains the same.

Symmetric to x axis: replace y with - y and relation remains the same.

Origin symmetry: replace x with - x, y with - y and the relation is equivalent.

63) $y = x^4 + x^2$	64) $y = \sin(x)$	65) $y = \cos(x)$
66) $y = \frac{-x}{x^5 + x^3}$	67) $x = y^2 + 1$	68) $y = \frac{1}{x-2}$

LOGARITHMIC AND EXPONENTIAL FUNCTIONS***Simplify Expressions.***

69) $\log_4 \left(\frac{1}{16} \right)$	70) $3\log_3 3 - \frac{3}{4}\log_3 81 + \frac{1}{3}\log_3 \left(\frac{1}{27} \right)$	71) $\log_9 27$	72) $\log_{125} \left(\frac{1}{5} \right)$
73) $\log_w w^{45}$	74) $\ln e$	75) $\ln 1$	76) $\ln e^2$

Solve equations.

77) $\log_6(x+3) + \log_6(x+4) = 1$	78) $\log x^2 - \log 100 = \log 1$	79) $3^{x+1} = 15$
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TRIGONOMETRY

Unit Circle – find the following without using a calculator.

80) $\sec\left(-\frac{\pi}{6}\right)$	81) $\tan\left(\frac{9\pi}{4}\right)$	82) $\cos\left(\frac{11\pi}{3}\right)$	83) $\sin\left(\frac{11\pi}{4}\right)$
84) $\cot 8\pi$	85) $\tan\left(\frac{5\pi}{2}\right)$	86) $\csc\left(\frac{-5\pi}{6}\right)$	87) $\sin\left(\frac{7\pi}{3}\right)$

State the domain, range, and fundamental period for each function.

88) $y = \sin x$	89) $y = \cos x$	90) $y = \tan x$
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Identities – simplify.

91) $\frac{(\tan^2 x)(\csc^2 x) - 1}{(\csc x)(\tan^2 x)(\sin x)}$	92) $1 - \cos^2 x$	93) $\sec^2 x - \tan^2 x$
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Solve equations.

94) $\cos^2 x = \cos x + 2$ $0 \leq x \leq 2\pi$	95) $2 \sin(2x) = \sqrt{3}$ $0 \leq x \leq 2\pi$
96) $4 \cos^2 x = 1$ $ x \leq \pi$	97) $\cos^2 x + \sin x + 1 = 0$ $-\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$

Inverse Trig Functions – Evaluate.

98) $\text{Arcsin}(1)$	99) $\text{Arcsin}\left(-\frac{\sqrt{2}}{2}\right)$	100) $\text{Arccos}\left(\frac{\sqrt{3}}{2}\right)$	101) $\text{in}\left(\text{Arccos}\left(\frac{\sqrt{3}}{2}\right)\right)$
102) State domain and range for: a) $\text{Arcsin}(x)$ b) $\text{Arccos}(x)$ c) $\text{Arctan}(x)$			

Graphing – State the amplitude and period of the following functions.

103) $y = -2\sin(2x)$	104) $y = 1/4\cos(4x + 12)$
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Geometry

Right Triangle Trig - Find the value of x.

105)

A right triangle with a vertical leg labeled x , a horizontal leg labeled 50 , and a hypotenuse labeled 10 . A right angle symbol is at the vertex between the legs.

106)

An isosceles triangle with a horizontal base labeled 10 . The two base angles are each labeled 70° . The two equal sides are labeled x .

107)

A diagram of a roller coaster track. It starts at point A, goes up an incline to point H, and then drops down to point C. A vertical line segment HB is drawn from point H to point B on the horizontal segment AC, with a right angle symbol at B. A roller coaster car is shown on the incline AH. The horizontal distance from A to B is 180 feet, and the horizontal distance from B to C is 60 feet.