

Calculus Summer Packet

Work the following problems in the space provided. Show all necessary work.

Algebra

Exponents.

$$1) \frac{(8x^3yz)^{\frac{1}{3}}(2x)^3}{4x^{\frac{1}{3}}(yz^{\frac{2}{3}})^{-1}}$$

Factor Completely.

$$2) 9x^2 + 3x - 3xy - y$$

$$3) 64x^6 - 1$$

$$4) 42x^4 + 35x^2 - 28$$

$$5) 15x^{\frac{5}{2}} - 2x^{\frac{3}{2}} - 24x^{\frac{1}{2}}$$

Rationalize the denominator / numerator.

$$6) \frac{x}{1-\sqrt{x-2}}$$

$$7) \frac{\sqrt{x+1}+1}{x}$$

Simplify the rational expression.

$$8) \frac{(x+1)^3(x-2)+3(x+1)^2}{(x+1)^4}$$

Solve algebraic equations and inequalities.

Use synthetic division to help factor the following, state all factors and roots.

9) $p(x) = x^3 + 4x^2 + x - 6$	10) $p(x) = 6x^3 - 17x^2 - 16x + 7$
11) $p(x) = 25x^3 + 50x^2 - 9x - 18$	12) $p(x) = x^4 - x^3 - 2x - 4$
13) <u>Explain</u> why $\frac{3}{2}$ cannot be a root of $4x^5 + cx^3 - dx + 5 = 0$, where c and d are integers.	14)) <u>Explain</u> why $x^4 + 7x^2 + x - 5 = 0$ must have a root in the interval $[0, 1]$, ($0 \leq x \leq 1$)

Solve.

15) $(x + 3)^2 > 4$	16) $\frac{x+5}{x-3} \leq 0$	17) $x^3 + 2x^2 - 3x \leq 0$
18) $\frac{x^2 - 9}{x+1} \geq 0$	19) $0 < x-2 < 8$	20) $ 5x-1 > 9$

Solve the system.

21) $x - y + 1 = 0$
 $y - x^2 = -5$

22) $x^2 - 4x + 3 = y$
 $-x^2 + 6x - 9 = y$

Graphing and Functions

Linear Graphs - Write the equation of the line described below.

23) Passes through the point (2, -1) and has slope -1/3.

24) Passes through the point (4, -3) and is perpendicular to $3x + 2y = 4$.

25) Passes through (-1, -2) and is parallel to $y = \frac{3}{5}x - 1$.

Conic Sections - Write the equation in standard form and identify the conic.

26) $x = 4y^2 + 8y - 3$

27) $x^2 + y^2 - 4x + 2y - 4 = 0$

28) $4x^2 - 16x + 3y^2 + 24y + 52 = 0$

29) $x^2 - 4y^2 + 2x + 24y - 19 = 0$

Functions

Find the domain and range of the following.

domain restrictions - denominator $\neq 0$, argument of a log or $\ln > 0$, radicand of even index must be ≥ 0
range restrictions- reasoning, if all else fails, use graphing calculator

30) $y = \frac{3}{x-2}$	31) $y = \log(x-3)$
32) $y = x^4 + x^2 + 2$	33) $y = \sqrt{2x-3}$
34) $y = x-5 $	35) Given $f(x)$ below, graph over the domain $[-3, 3]$. What is the range? $f(x) = \begin{cases} x & \text{if } x \geq 0 \\ 1 & \text{if } -1 \leq x < 0 \\ x-2 & \text{if } x < -1 \end{cases}$

Compositions and Inverses - Find the compositions and inverses as indicated below.

Let:	$f(x) = x^2 + 3x - 2$	$g(x) = 4x - 3$	$h(x) = \ln x$	$w(x) = \sqrt{x-4}$
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36) $g^{-1}(x)$	37) $h^{-1}(x)$	38) $w^{-1}(x)$, for $x \geq 4$	39) $f(g(x))$
40) $h(g(f(1)))$	41) $g(h(x))$	42) $g(w(x))$	43) Does $y = 3x^2 - 9$ have an inverse function? Explain your answer.

Basic Shapes of Curves:

Sketch the graphs on separate paper. You may use your graphing calculator to verify the graph, but you should be able to graph the following by knowledge of the shape of the curve, by plotting a few points, and by your knowledge of transformations.

44) $y = \sqrt{x}$	45) $y = \ln x$	46) $y = x - 2 $
47) $y = \frac{1}{x}$	48) $y = \frac{1}{x-2}$	49) $y = \frac{x}{x^2 - 4}$
50) $y = y = e^x$	51) $y = 3^x$	52) $f(x) = \begin{cases} \sqrt{25-x^2} & \text{if } x < 0 \\ \frac{x^2-25}{x-5} & \text{if } x \geq 0, x \neq 5 \\ 0 & \text{if } x = 5 \end{cases}$

Asymptotes – Identify any asymptotes.

53) $y = \frac{1}{x-1}$	54) $x^2 - y^2 = 1$	55) $y = \frac{x^2}{x^2 - 1}$
56) $y = \frac{3x^2}{2x^2 - 3x + 3}$	57) $y = \frac{5x^2 - 5x + 1}{x - 1}$	58) $y = \frac{2x^2}{3x^3 - 4x + 1}$

Even and Odd Functions - Identify as odd, even, or neither. Show substitutions!

Even: $f(x) = f(-x)$ Odd: $f(-x) = -f(x)$

59) $f(x) = x^3 + 3x$	60) $f(x) = x^4 - 6x^2 + 3$	61) $f(x) = \frac{x^3 - x}{x^2}$	62) $f(x) = \sin 2x$
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Test for symmetry. Show substitutions.

Symmetric to y axis: replace x with -x and relation remains the same.

Symmetric to x axis: replace y with -y and relation remains the same.

Origin symmetry: replace x with -x, y with -y and the relation is equivalent.

63) $y = x^4 + x^2$	64) $y = \sin(x)$	65) $y = \cos(x)$
66) $y = \frac{-x}{x^5 + x^3}$	67) $x = y^2 + 1$	68) $y = \frac{1}{x-2}$

LOGARITHMIC AND EXPONENTIAL FUNCTIONS***Simplify Expressions.***

69) $\log_4\left(\frac{1}{16}\right)$	70) $3\log_3 3 - \frac{3}{4}\log_3 81 + \frac{1}{3}\log_3\left(\frac{1}{27}\right)$	71) $\log_9 27$	72) $\log_{125}\left(\frac{1}{5}\right)$
73) $\log_w w^{45}$	74) $\ln e$	75) $\ln 1$	76) $\ln e^2$

Solve equations.

77) $\log_6(x+3) + \log_6(x+4) = 1$	78) $\log x^2 - \log 100 = \log 1$	79) $3^{x+1} = 15$
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TRIGONOMETRY

Unit Circle – find the following without using a calculator.

80) $\sec\left(-\frac{\pi}{6}\right)$	81) $\tan\left(\frac{9\pi}{4}\right)$	82) $\cos\left(\frac{11\pi}{3}\right)$	83) $\sin\left(\frac{11\pi}{4}\right)$
84) $\cot 8\pi$	85) $\tan\left(\frac{5\pi}{2}\right)$	86) $\csc\left(\frac{-5\pi}{6}\right)$	87) $\sin\left(\frac{7\pi}{3}\right)$

State the domain, range, and fundamental period for each function.

88) $y = \sin x$	89) $y = \cos x$	90) $y = \tan x$
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Identities – simplify.

91) $\frac{(\tan^2 x)(\csc^2 x) - 1}{(\csc x)(\tan^2 x)(\sin x)}$	92) $1 - \cos^2 x$	93) $\sec^2 x - \tan^2 x$
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Solve equations.

94) $\cos^2 x = \cos x + 2$ $0 \leq x \leq 2\pi$	95) $2 \sin(2x) = \sqrt{3}$ $0 \leq x \leq 2\pi$
96) $4 \cos^2 x = 1$ $ x \leq \pi$	97) $\cos^2 x + \sin x + 1 = 0$ $-\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$

Inverse Trig Functions – Evaluate.

98) $\text{Arcsin}(1)$

99) $\text{Arcsin}\left(-\frac{\sqrt{2}}{2}\right)$

100) $\text{Arccos}\left(\frac{\sqrt{3}}{2}\right)$

101) $\text{in}\left(\text{Arccos}\left(\frac{\sqrt{3}}{2}\right)\right)$

102) State domain and range for:

a) $\text{Arcsin}(x)$

b) $\text{Arccos}(x)$

c) $\text{Arctan}(x)$

Graphing – State the amplitude and period of the following functions.

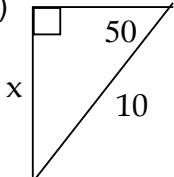
103) $y = -2\sin(2x)$

104) $y = 1/4\cos(4x + 12)$

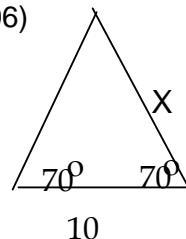
Geometry

Right Triangle Trig - Find the value of x.

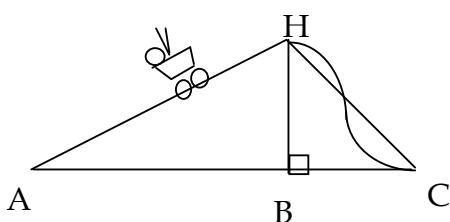
105)



106)



107)



The roller coaster car shown in the diagram above takes 23.5 sec. to go up the 23 degree incline segment AH and only 2.8 seconds to go down the drop from H to C. The car covers horizontal distances of 180 feet on the incline and 60 feet on the drop.

- How high is the roller coaster above point B?
- Find the distances AH and HC.
- How fast (in ft/sec) does the car go up the incline?
- What is the approximate average speed of the car as it goes down the drop?
(Assume the car travels along HC.)
- Is your approximate answer too big or too small?
(Advanced Mathematics, Richard G. Brown, Houghton Mifflin, 1994, pg 336)